

式の展開と因数分解⑦ (解答と解説)

[1] [解答] (1) $2a + 3b$

$$(2) -8x^2 + 10xy - 3y^2 \quad (3) y^2 - y - 30$$

$$(4) x^2 + 16x + 64 \quad (5) 1 - p^2$$

$$(6) 4x^2 + y^2 - 4z^2 + 4xy + 3yz + 6zx$$

$$\begin{aligned} (1) \quad (8a^2 + 12ab) \div 4a &= (8a^2 + 12ab) \times \frac{1}{4a} \\ &= \frac{8a^2}{4a} + \frac{12ab}{4a} \\ &= 2a + 3b \end{aligned}$$

$$\begin{aligned} (2) \quad (-4x + 3y)(2x - y) &= -8x^2 + 4xy + 6xy - 3y^2 \\ &= -8x^2 + 10xy - 3y^2 \end{aligned}$$

$$\begin{aligned} (3) \quad (y + 5)(y - 6) &= y^2 + [5 + (-6)]y + 5 \times (-6) \\ &= y^2 - y - 30 \end{aligned}$$

$$\begin{aligned} (4) \quad (x + 8)^2 &= x^2 + 2 \times 8 \times x + 8^2 \\ &= x^2 + 16x + 64 \end{aligned}$$

$$\begin{aligned} (5) \quad (-p + 1)(1 + p) &= (1 - p)(1 + p) \\ &= 1^2 - p^2 \\ &= 1 - p^2 \end{aligned}$$

$$\begin{aligned} (6) \quad (2x + y - z)(2x + y + 4z) &= [(2x + y) - z][(2x + y) + 4z] \\ &= (2x + y)^2 + 3z(2x + y) - 4z^2 \\ &= 4x^2 + 4xy + y^2 + 6zx + 3yz - 4z^2 \\ &= 4x^2 + y^2 - 4z^2 + 4xy + 3yz + 6zx \end{aligned}$$

[2] [解答] (1) $3ax(x + 3a - 5)$ (2) $(x - 4)(x - 6)$ (3) $(x + 1)(x - 8)$

$$(4) (a + 9)^2 \quad (5) (3 + t)(3 - t) \quad (6) (x + y + 3)(x - y + 3)$$

$$\begin{aligned} (1) \quad 3ax^2 + 9a^2x - 15ax &= 3ax \times x + 3ax \times 3a - 3ax \times 5 \\ &= 3ax(x + 3a - 5) \end{aligned}$$

$$\begin{aligned} (2) \quad x^2 - 10x + 24 &= x^2 + [(-4) + (-6)]x + (-4) \times (-6) \\ &= (x - 4)(x - 6) \end{aligned}$$

$$(3) \quad x^2 - 7x - 8 = (x + 1)(x - 8)$$

$$\begin{aligned} (4) \quad a^2 + 18a + 81 &= a^2 + 2 \times 9 \times a + 9^2 \\ &= (a + 9)^2 \end{aligned}$$

$$(5) \quad 9 - t^2 = 3^2 - t^2$$

$$= (3 + t)(3 - t)$$

$$[9 - t^2 = -(t^2 - 9) = -(t + 3)(t - 3) \text{ としてもよい}]$$

$$\begin{aligned} (6) \quad x^2 + 6x + 9 - y^2 &= (x^2 + 6x + 9) - y^2 \\ &= (x + 3)^2 - y^2 \\ &= [(x + 3) + y][(x + 3) - y] \\ &= (x + y + 3)(x - y + 3) \end{aligned}$$

[3] [解答] (1) 160 (2) 2000

$$\begin{aligned} (1) \quad 22^2 - 18^2 &= (22 + 18)(22 - 18) \\ &= 40 \times 4 \\ &= 160 \end{aligned}$$

$$\begin{aligned} (2) \quad 105^2 - 95^2 &= (105 + 95)(105 - 95) \\ &= 200 \times 10 \\ &= 2000 \end{aligned}$$

[4] [解答] 略

M は、正の奇数 N を N 個加えた和であるから $M = N \times N$
 n を整数として、 $N = 2n + 1$ と表すと

$$\begin{aligned} M - 1 &= (2n + 1)(2n + 1) - 1 \\ &= 4n^2 + 4n + 1 - 1 \\ &= 4(n^2 + n) \end{aligned}$$

$n^2 + n$ は整数であるから、 $4(n^2 + n)$ は 4 の倍数である。
よって、 $M - 1$ は 4 の倍数である。