

[1] 解答

$$\begin{aligned} (1) \quad & x^2 + y^2 + z^2 - 2xy - 2yz + 2zx \\ &= 25x^2 - 10xy + y^2 + 30zx + 9z^2 \\ &= 25x^2 + y^2 + 9z^2 - 10xy - 6yz + 30zx \end{aligned}$$

(2) $a^2 + b^2 + 2ab + 6a + 6b + 9$

(3) $a^2 + b^2 + 4c^2 + 2ab + 4bc + 4ca$

(4) $9a^2 + 4b^2 + 16c^2 + 12ab - 16bc - 24ca$

(5) $4p^2 + 9q^2 + r^2 - 12pq - 6qr + 4rp$

(6) $25x^2 + y^2 + 9z^2 - 10xy - 6yz + 30zx$

(1) $(x-y+z)^2 = [(x-y)+z]^2$

$$\begin{aligned} &= (x-y)^2 + 2(x-y)z + z^2 \\ &= x^2 - 2xy + y^2 + 2zx - 2yz + z^2 \\ &= x^2 + y^2 + z^2 - 2xy - 2yz + 2zx \end{aligned}$$

(2) $(a+b+3)^2 = [(a+b)+3]^2$

$$\begin{aligned} &= (a+b)^2 + 2(a+b) \times 3 + 3^2 \\ &= a^2 + 2ab + b^2 + 6a + 6b + 9 \\ &= a^2 + b^2 + 2ab + 6a + 6b + 9 \end{aligned}$$

(3) $(a+b+2c)^2 = [(a+b)+2c]^2$

$$\begin{aligned} &= (a+b)^2 + 2(a+b) \times 2c + (2c)^2 \\ &= a^2 + 2ab + b^2 + 4ca + 4bc + 4c^2 \\ &= a^2 + b^2 + 4c^2 + 2ab + 4bc + 4ca \end{aligned}$$

(4) $(3a+2b-4c)^2 = [(3a+2b)-4c]^2$

$$\begin{aligned} &= (3a+2b)^2 - 2(3a+2b) \times 4c + (4c)^2 \\ &= 9a^2 + 12ab + 4b^2 - 24ca - 16bc + 16c^2 \\ &= 9a^2 + 4b^2 + 16c^2 + 12ab - 16bc - 24ca \end{aligned}$$

(5) $(2p-3q+r)^2 = [(2p-3q)+r]^2$

$$\begin{aligned} &= (2p-3q)^2 + 2(2p-3q) \times r + r^2 \\ &= 4p^2 - 12pq + 9q^2 + 4rp - 6qr + r^2 \\ &= 4p^2 + 9q^2 + r^2 - 12pq - 6qr + 4rp \end{aligned}$$

(6) $(-5x+y-3z)^2 = [(-5x+y)-3z]^2$

$$\begin{aligned} &= (-5x+y)^2 - 2(-5x+y) \times 3z + (3z)^2 \\ &= (-5x+y)^2 - 2(-5x+y) \times 3z + (3z)^2 \end{aligned}$$

[2]

(1) $x^2 + 2xy + y^2 + 5x + 5y + 6$
(2) $a^2 + 2ab + b^2 + 7a + 7b - 60$

(3) $x^2 - 4xy + 4y^2 - 6x + 12y - 27$

(4) $4a^2 - 12ab + 9b^2 - 6a + 9b - 4$

(5) $4x^2 + y^2 - 4z^2 + 4xy + 3yz + 6zx$

(6) $16p^2 + 25q^2 - 6r^2 - 40pq + 5qr - 4rp$

(7) $7x^2 + 9y^2 + 16z^2 + 24xy - 24yz - 32zx$

(8) $28a^2 + 9b^2 + 4c^2 - 33ab + 12bc - 22ca$

(1) $(x+y+3)(x+y+2) = \{(x+y)+3\} \{(x+y)+2\}$

$= (x+y)^2 + 5(x+y) + 6$

(2) $(a+b-5)(a+b+12) = \{(a+b)-5\} \{(a+b)+12\}$

$= (a+b)^2 + 7(a+b) - 60$

$= a^2 + 2ab + b^2 + 7a + 7b - 60$

(3) $(x-2y+3)(x-2y-9) = \{(x-2y)+3\} \{(x-2y)-9\}$

$= (x-2y)^2 - 6(x-2y) - 27$

(4) $(2a-3b+1)(2a-3b-4) = \{(2a-3b)+1\} \{(2a-3b)-4\}$

$= (2a-3b)^2 - 3(2a-3b) - 4$

$= 4a^2 - 12ab + 9b^2 - 6a + 9b - 4$

(5) $(2x+y-z)(2x+y+4z) = \{(2x+y)-z\} \{(2x+y)+4z\}$

$= (2x+y)^2 + 3z(2x+y) - 4z^2$

(6) $(4p-5q+2r)(4p-5q-3r) = \{(4p-5q)+2r\} \{(4p-5q)-3r\}$

$$\begin{aligned} &= (4p-5q)^2 - r(4p-5q) - 6r^2 \\ &= 16p^2 - 40pq + 25q^2 - 4rp + 5qr - 6r^2 \\ &= 16p^2 + 25q^2 - 6r^2 - 40pq + 5qr - 4rp \\ &= 7x^2 + 8x(3y-4z) + (3y-4z)^2 \\ &= 7x^2 + 24xy - 32zx + 9y^2 - 24yz + 16z^2 \end{aligned}$$

(8) $(4a-3b-2c)(7a-3b-2c) = \{4a-(3b+2c)\} \{7a-(3b+2c)\}$
 $= 28a^2 - 11a(3b+2c) + (3b+2c)^2$
 $= 28a^2 - 33ab - 22ca + 9b^2 + 12bc + 4c^2$
 $= 28a^2 + 9b^2 + 4c^2 - 33ab + 12bc - 22ca$

$$\begin{aligned} &= 7x^2 + 9y^2 + 30yz - 25z^2 \\ &= (2) \quad x^2 - 9y^2 + 30yz - 25z^2 \\ &= (3) \quad 3x^2 + 24xy + 48y^2 - 7x - 28y + 2 \\ &= (4) \quad a^4 + 15a^3 + 26a^2 - 48a + 16 \end{aligned}$$

3 [解答] (1) $a^2 - b^2 + 2bc - c^2$
(2) $x^2 - 9y^2 + 30yz - 25z^2$
(3) $3x^2 + 24xy + 48y^2 - 7x - 28y + 2$
(4) $a^4 + 15a^3 + 26a^2 - 48a + 16$

(1) $(a+b-c)(a-b+c) = \{a+(b-c)\} \{a-(b-c)\}$

$= a^2 - (b-c)^2$

$= a^2 - (b^2 - 2bc + c^2)$

$= a^2 - b^2 + 2bc - c^2$

(2) $(x-3y+5z)(x+3y-5z) = \{x-(3y-5z)\} \{x+(3y-5z)\}$

$= x^2 - (3y-5z)^2$

$= x^2 - (9y^2 - 30yz + 25z^2)$

$= x^2 - 9y^2 + 30yz - 25z^2$

(3) $(x+4y-2)(3x+12y-1) = \{(x+4y)-2\} \{3(x+4y)-1\}$

$= 3(x+4y)^2 - 7(x+4y) + 2$

(4) $(x-3y+8xy+16y^2)^2 - 7x - 28y + 2$

$= 3x^2 + 24xy + 48y^2 - 7x - 28y + 2$

(5) $(a^2 + 3a - 2)(a^2 + 12a - 8) = \{a^2 + (3a-2)\} \{a^2 + 4(3a-2)\}$

$= (a^2)^2 + 5(3a-2) \times a^2 + 4(3a-2)^2$

$= a^4 + 15a^3 - 10a^2 + 4(9a^2 - 12a + 4)$

$= a^4 + 15a^3 - 10a^2 + 36a^2 - 48a + 16$

$= a^4 + 15a^3 + 26a^2 - 48a + 16$